SET-3

Series : 1PQRS

प्रश्न-पत्र कोड
Q.P. Code **65/1/3**

रोल नं.
Roll No. | 2 | 1 | 6 | 5 | 6 | 9 | 9 | 1 |

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **23** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the serial number of the question in the answer-book at the given place before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. {}

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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in few questions in all the Sections except Section – **A**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

Question **1** to **20** are multiple choice questions of **1** mark each.

1. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$ 1

   (A) $6\sqrt{3}$ (B) $8\sqrt{3}$

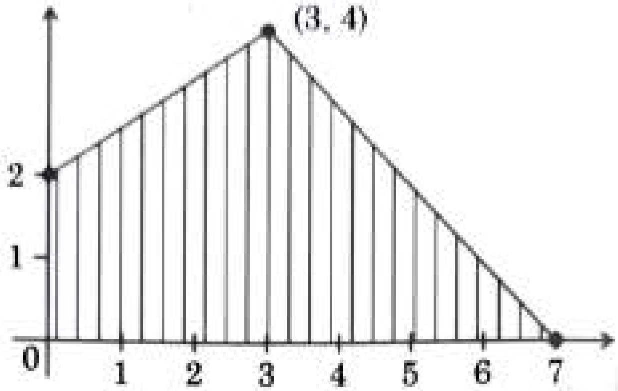
   (C) $12\sqrt{3}$ (D) $3\sqrt{12}$

2. The length of perpendicular drawn from the point (3, 4, 2) on the line $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is 1

   (A) 2 (B) 9

   (C) 5 (D) $\sqrt{29}$

3. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below : 1



The maximum value of Z – minimum value of Z is

(A) 8 (B) 29

(C) 35 (D) 43

4. The degree of an objective function of a linear programming problem is 1

(A) 0 (B) 1

(C) 2 (D) Any natural number

5. If $\sin^{-1}x + \pi = y$, then 1

(A) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (B) $-\frac{3\pi}{2} \leq y \leq -\frac{\pi}{2}$

(C) $\frac{\pi}{2} \leq y \leq \frac{3\pi}{2}$ (D) $0 \leq y \leq \pi$

6. If $A = [a_{ij}]_{3 \times 3}$ is a scalar matrix then which of the following must be true ? 1

(A) A must be a symmetric matrix.

(B) A must be a skew-symmetric matrix.

(C) A must be an identity matrix.

(D) A must be a null matrix.

7. Which of the following properties is/are true for two matrices of suitable orders ? 1

(i) $(A + B)' = A' + B'$

(ii) $(A - B)' = B' - A'$

(iii) $(AB)' = A'B'$

(iv) $(kAB)' = kB'A'$ (k is a scalar)

(A) (i) only

(B) (i), (ii) and (iii)

(C) (i) and (ii)

(D) (i) and (iv) •

8. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then 1

(A) $\Delta_1 = 2\Delta_2$

(B) $\Delta_2 = -2\Delta_1$ •

(C) $\Delta_1 = \Delta_2$

(D) $\Delta_2 = -\Delta_1$

9. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is 1

(A) 0

(B) $\frac{\pi}{4}$ •

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

10. If A and B are symmetric matrics of same order, then (AB – BA) is a 1

(A) Zero matrix

(B) Identity matrix

(C) Symmetric matrix

(D) Skew symmetric matrix •

11. The least value of $f(x) = e^{-x}$ in [0, 3] is 1

(A) e^{-3} •

(B) -1

(C) 1

(D) $-e^{3}$

12. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log |b^2 + c^2x^2| + K$, then the value of A is 1

(A) 3a

(B) $\frac{3a}{2b^2}$

(C) $\frac{3a}{b^2c^2}$

(D) $\frac{3a}{2c^2}$ •

13. The value of $\int_{-1}^{1} \frac{x^3}{x^2 + 2|x| + 1} dx$ is 1

(A) 0
(B) $\log 2$
(C) $2 \log 2$
(D) $\frac{1}{2} \log 2$

14. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by 1

(A) 0
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{4}{3}$

15. The integrating factor of differential equation $R\frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is 1

(A) $e^{\int \frac{P}{Q} dy}$
(B) $e^{\int P dy}$
(C) $e^{\int \frac{P}{R} dy}$
(D) $e^{\int \frac{P}{R} dx}$

16. The order and degree of the differential equation : 1

$\frac{d}{dx}(y')^3 + (y')^3 = 1$ respectively are where $y' = \frac{dy}{dx}$

(A) 1, 3
(B) 2, 1
(C) 3, 1
(D) 3, 2

17. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is 1

(A) 0
(B) 1
(C) $\frac{5}{2}$
(D) $-\frac{5}{2}$

18. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is 1

(A) 8

(B) -8

(C) 2

(D) $\frac{5}{2}$

Assertion – Reason Based Questions

Direction : Question numbers **19** and **20** are Assertion and Reason based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true and Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A) :** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$. c 1

Reason (R) : For any two events A and B, $P(A|B) = \frac{P(A \cup B)}{P(B)}$

20. **Assertion (A) :** Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$. 1

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$. D

SECTION – B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Simplify : $\tan^{-1}\left(\dfrac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 < x < \dfrac{\pi}{4}$. 2

OR

(b) Evaluate : $\tan\left(\sin^{-1} 1 - \cos^{-1}\left(-\dfrac{1}{2}\right)\right)$ 2

22. (a) Check whether function $f(x)$ defined as 2

$$f(x) = \begin{cases} \dfrac{|x-3|}{2(x-3)}, & x < 3 \\ \dfrac{x-6}{6}, & x \geq 3 \end{cases}$$ is continuous at $x = 3$ or not ?

OR

(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$. 2

23. (a) Simplify : $\sin^{-1}\sqrt{\dfrac{1 + \cos 2x}{2}}$, $0 < x < \dfrac{\pi}{2}$. 2

OR

(b) Evaluate : $\cos[\sin^{-1}(-1) - \tan^{-1}(-\sqrt{3})]$. 2

24. Using vectors, find the area of ΔABC with vertices A(1, 2, 3), B(2, –1, 4) and C(4, 5, –1). 2

25. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths. 2

$1(\hat{i} - \hat{j})$

$4\hat{i} - 2\hat{j} + 4\hat{k}$

SECTION - C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. Evaluate : $\int_0^1 \log(1 + x^2)dx$ 3

27. (a) Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball. 3

OR

(b) The probability of simultaneous occurrence of atleast one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$. 3

28. (a) Find $\int \sqrt{\frac{x+2}{x-2}}\,dx$ 3

OR

(b) Find : $\int \frac{x^2}{(x^2+9)(x^2+16)}\,dx$ 3

29. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x|\,dx$, then show that $I_1 - 4I_2 = 0$. 3

30. (a) Find the general solution of the differential equation 3

$(y^2 - x^2)dx = 2xy\,dy$

OR

(b) Find the particular solution of the differential equation

$(1 + e^{2x})dy + (1 + y^2)e^x\,dx = 0$, given that $y(1) = 0$. 3

31. Solve the following linear programming problem graphically : 3

Minimize $Z = 13x - 15y$

Subject to constraints

$$x + y \le 7,$$

$$2x - 3y + 6 \ge 0,$$

$$x \ge 0, y \ge 0$$

SECTION – D

This section comprises Long Answer (LA) type questions of **5** marks each.

32. Show that line AB passing through points A(0, 4, 1), B(2, 3, −1) and the line CD passing through points C(4, 5, 0), D(2, 6, 2) are parallel. Also, find distance between them. 5

33. (a) A relation R is defined on Z, the set of integers, as 5

$R = \{(x, y) : |x - y| \text{ is divisible by a prime number 'p'}, x, y \in Z\}$

check whether R is an equivalence relation or not.

OR

(b) A function $f : R - \left\{\frac{3}{5}\right\} \longrightarrow R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x + 2}{5x - 3}$. Show that f is one-one and onto. 5

34. (a) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$ 5

OR

(b) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x

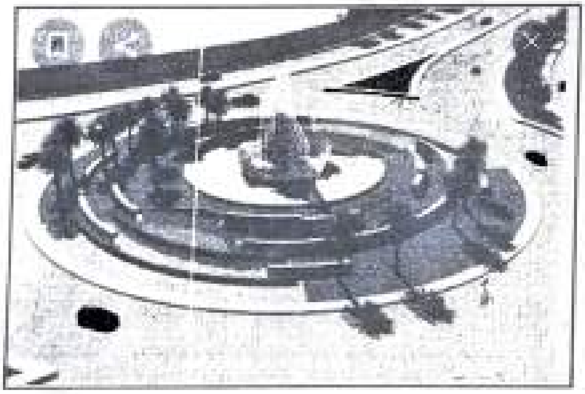
where $x \in \left[0, \frac{\pi}{2}\right]$. 5

35. If $x = a(\sin t - t\cos t)$ and $y = b(\cos t + t\sin t)$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. 5

SECTION – E

This section comprises of **3** case study based questions of **4** marks each.

36. Roundabouts are often made on busy roads to ease the traffic and avoid red lights.



One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$.

There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Based on the given information, answer the following questions :

(i) Represent the given equations C_1 and C_2 with the help of a diagram. 1

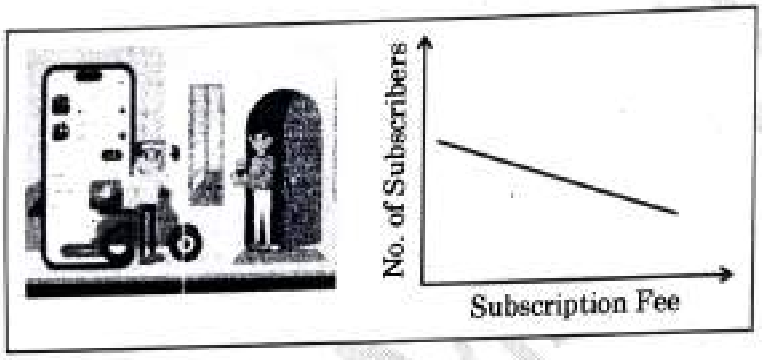
(ii) Express y as a function of x, ($y = f(x)$), for both C_1 an C_2. 1

(iii) (a) Using integration find the area of region covered by the roundabout. 2

OR

(iii) (b) Using integration, find the area of region covered by circular pond. 2

37. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.



The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1, ten subscribers will discontinue. Assume that the company increased the annual fee by ₹ x.

Based on the given information, answer the following questions :

(i) How many subscribers will discontinue after an increase of ₹ x in annual fee ? 1

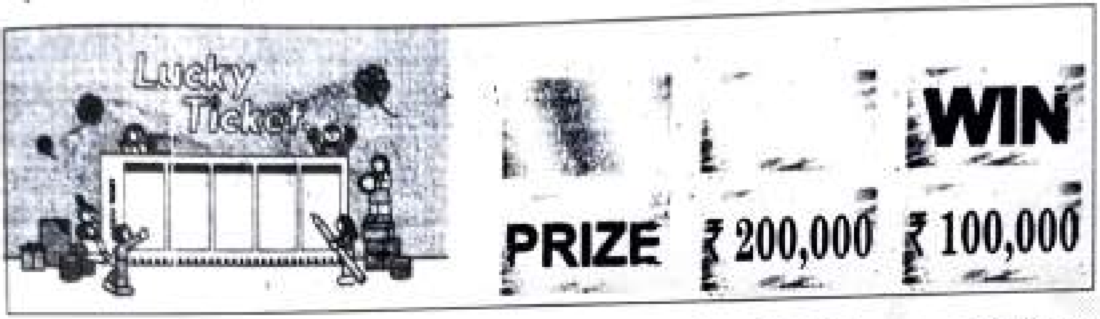
(ii) If $R(x)$ denotes the total revenue collected after the increase of ₹ x in subscription fee, express $R(x)$ as a function of x. 1

(iii) (a) Find the value of x for which $R(x)$ is maximum. 2

OR

(iii) (b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing. 2

38. In an online jackpot, there is one first prize of ₹ 3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.



A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000.

Rohan bought one ticket.

Based on given information, answer the following questions :

(i) What are the possible amounts, the person can win ? 1

(ii) (a) What is the probability that the person wins atleast ₹ 2,00,000 ? 2

OR

(ii) (b) What is the probability that the person does not win any amount ? 2

(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot. 1